## TABLEAUX ON PERIODIC SKEW DIAGRAMS AND IRREDUCIBLE REPRESENTATIONS OF THE DOUBLE AFFINE HECKE ALGEBRA OF TYPE $\cal A$

## TAKESHI SUZUKI AND MONICA VAZIRANI\*

ABSTRACT. We introduce and study an affine analogue of skew Young diagrams and tableaux on them.

It turns out that the double affine Hecke algebra of type A acts on the space spanned by standard tableaux on each diagram. It is shown that the modules obtained this way are irreducible, and they exhaust all irreducible modules of a certain class over the double affine Hecke algebra. In particular, the classification of irreducible modules of this class, announced by Cherednik, is recovered.

## 1. Introduction

As is well-known, Young diagrams consisting of n boxes parameterize isomorphism classes of finite dimensional irreducible representations of the symmetric group  $\mathfrak{S}_n$  of degree n, and moreover the structure of each irreducible representation is described in terms of tableaux on the corresponding Young diagram; namely, a basis of the representation is labeled by standard tableaux, with which the action of  $\mathfrak{S}_n$  generators is explicitly described. This combinatorial description due to A. Young has played an essential role in the study of the representation theory of the symmetric group (or the affine Hecke algebra), and its generalization to the (degenerate) affine Hecke algebra  $H_n(q)$  of  $GL_n$  has been given in [Ch1, Ra1, Ra2], where skew Young diagrams appear on combinatorial side.

The purpose of this paper is to introduce an "affine analogue" of skew Young diagrams and tableaux, which give a parameterization and a combinatorial description of a family of irreducible representations of the double affine Hecke algebra  $\ddot{H}_n(q)$  of  $GL_n$  over a field  $\mathbb{F}$ , where  $q \in \mathbb{F}$  is a parameter of the algebra.

The double affine Hecke algebra was introduced by I. Cherednik [Ch2, Ch3] and has since been used by him and by several authors to obtain important results about diagonal coinvariants, Macdonald polynomials, and certain Macdonald identities.

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In this paper, we focus on the case where q is not a root of 1, and we consider representations of  $\ddot{H}_n(q)$  that are  $\mathfrak{X}$ -semisimple; namely, we consider representations which have basis of simultaneous eigenvectors with respect to all elements in the commutative subalgebra  $\mathbb{F}[\mathfrak{X}] = \mathbb{F}[x_1^{\pm 1}, x_2^{\pm 1}, \dots, x_n^{\pm 1}, \xi^{\pm 1}]$  of  $\ddot{H}_n(q)$ . (In [Ra1, Ra2], such representations for affine Hecke algebras are referred to as "calibrated")

On combinatorial side, we introduce *periodic skew diagrams* as skew Young diagrams consisting of infinitely many boxes satisfying certain periodicity conditions. We define a tableau on a periodic skew diagram as a bijection from the diagram to  $\mathbb{Z}$  which satisfies the condition reflecting the periodicity of the diagram.

Periodic skew diagrams are natural generalization of skew Young diagrams and have appeared in [Ch4] (or implicitly in [AST]), but the notion of tableaux on them seems new.

To connect the combinatorics with the representation theory of the double affine Hecke algebra  $\ddot{H}_n(q)$ , we construct, for each periodic skew diagram, an  $\ddot{H}_n(q)$ -module that has a basis of  $\mathbb{F}[\mathfrak{X}]$ -weight vectors labeled by standard tableaux on the diagram by giving the explicit action of the  $\ddot{H}_n(q)$  generators.

Such modules are  $\mathfrak{X}$ -semisimple by definition. We show that they are irreducible, and that our construction gives a one-to-one correspondence between the set of periodic skew diagrams and the set of isomorphism classes of irreducible representations of the double affine Hecke algebra that are  $\mathfrak{X}$ -semisimple.

The classification results here recover those of Cherednik's in [Ch4] (see also [Ch5]), but in this paper we provide a detailed proof based on purely combinatorial arguments concerning standard tableaux on periodic skew diagrams.

Note that the corresponding results for the degenerate double affine Hecke algebra of  $GL_n$  easily follow from a parallel argument.

An outline of the paper is as follows. Section 2 is a review of the affine root system and the extended affine Weyl group of  $\widehat{\mathfrak{gl}}_n$ .

The contents of Section 3 are purely combinatorial. We introduce periodic skew diagrams and tableaux on them in Section 3.1 and Section 3.2 respectively. These combinatorial objects are considered worth studying in themselves, and here we investigate their relation with the affine Weyl group and *content* functions. The set of tableaux on a periodic skew diagram admits an action of the extended affine Weyl group  $\dot{W}$ , and it turns out that this action is simply transitive and gives a bijective correspondence between the tableaux and the elements of  $\dot{W}$ . In Section 3.5, we explicitly describe the subset  $\dot{W}$  corresponding to

wherein the automatic color matching mode processing unit comprises a pre-printing correction mode, a periodic correction mode, and a composite correction mode including the pre-printing correction mode and the periodic correction mode, any one of which is selected by operator's operation.

## Claim 3 (canceled).

Claim 4 (currently amended): The device according to claim 3, wherein when the automatic color matching mode processing unit recognizes selection of the pre-printing correction mode by the operator, [[this]] said automatic color matching mode processing unit [[works]] controls the color matching processing unit before start of printing in the case of receipt of a printing request.

Claim 5 (currently amended): The device according to claim 3, wherein when the automatic color matching mode processing unit recognizes selection of the periodic correction mode by the operator, [[this]] said automatic color matching mode processing unit [[works]] controls the color matching processing unit, in a printing wait state, whenever elapsed time Tw from the preceding color matching processing reaches a given periodic time n.

Claim 6 (currently amended): The device according to claim 3, wherein in the case that the automatic color matching mode processing unit recognizes selection of the composite correction mode by the operator, at the time of receiving a printing request [[this]] said automatic color matching mode processing unit [[works]] controls the color matching processing unit before start of printing; and [[this]] said automatic color matching mode processing unit [[works]] controls the color matching processing unit, in a printing wait state, whenever elapsed time Tw from the preceding color matching processing reaches a given periodic time n.

Claim 7 (currently amended): The device according to claim 4, wherein in the case that the automatic color matching mode processing unit recognizes selection of the pre-printing correction mode by the operator, [[this]] said automatic color matching mode processing unit starts printing without working controlling the color matching processing unit when said automatic color matching mode processing unit receives the printing request and elapsed time. Two from the preceding color matching processing is below a given time m; and when the elapsed time Two is not less than the given time m, the automatic color matching mode processing unit [[works]] controls the color matching processing unit and subsequently starts printing.

Claim 8 (currently amended): The device according to claim 1, wherein when the manual color matching mode processing unit recognizes the operator's manual color matching instructing operation, [[this]] said manual color matching mode processing unit [[works]]

<u>controls</u> the color matching processing unit forcibly even if the automatic mode of the automatic color matching mode processing unit is selected.

Claim 9 (original): The device according to claim 1, wherein the color matching processing unit performs density correction of the respective color images designated by the operator, as well as color slippage correction of the different color images.

Claim 10 (original): The device according to claim 1, which comprises an operator operation panel for performing mode selection operation for the automatic color matching mode processing unit, and manual color matching designation operation for the manual color matching processing unit.

Claim 11 (currently amended): The device according to claim 1, which comprises an interface processing unit which perform performs a mode selection operation for the automatic color matching mode processing unit through a screen of a terminal of an external unit connected to a network, and receive receives and process processing a request of manual color matching designation operation for the manual color matching processing unit.